High Density Polyethylene Foams. IV. Flexural and Tensile Moduli of Structural Foams

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ABSTRACT: High density closed-cell polyethylene foams (450–950 kg/m^3) were prepared by compression molding, and their flexural and tensile moduli were measured in order to study (1) the normalized modulus as a function of the normalized density, and (2) the effect of thin skins on flexural and tensile moduli. For the flexural data, it was found that the model of Gonzalez and the I-beam model of Hobbs predicted the data very well in the range of void volume fractions under study (0–55%). For the tensile data, it was found that a combination of the differential scheme or the square power-law model with the sandwich structure gave the best predictions. Finally, we found that thin skins have an important effect on the flexural properties of polymer foams, while they seem to have a negligible effect on the tensile properties. © 2003 Wiley Periodicals, Inc. J Appl Polym Sci 90: 2139–2149, 2003

Key words: polyethylene; mechanical properties; modulus

INTRODUCTION

Flexural and tensile properties of high density polymer foams are important for the design of structural materials. This is the last paper in this series on the mechanical and morphological properties of high density polyethylene (HDPE) foams.[1–3] In the second part of this series,[2] we compared Young's modulus with different models for uniform foams. Of all the models used in our comparison, it was found that the differential scheme and Moore's simple empirical equation[4] gave similar results and were best at predicting the data in the range of void volume fractions under study (0–55%). In this paper, a focus is made on the properties of uniform and structural foams, which are composed of foamed cores enclosed by unfoamed skins.

It is known that the elastic moduli are different for different types of deformation (flexural vs. tensile) and measurement conditions. The flexural and tensile properties of foams are now investigated and compared with available models from the literature. After careful microscopic measurement, our HDPE foams were found to have very thin, but not negligible, skin layers on both sides. It is thus the objective of this paper to determine the effect of these skin layers on the flexural and tensile moduli of these structural foams. Flexural moduli of structural foams have been the subject of several reports. Most of the literature on the subject of the flexural moduli of polymer foams relates the stiffness of a rectangular beam to the density of the foam.[5–12] Hartsock[5] assumed that the stiffness of a sandwich structure is only related to the contribution of the skin layers. From that, the maximum deflection, maximum face stress, and maximum core shear stress were obtained. Throne[6,7] suggested that for structural foams, the flexural modulus was the average modulus through integration across the foam thickness from a relationship between the local modulus and the local density. However, such a dependence of local density on position is difficult to measure. Gonzalez[8] assumed that the thickness of the intermediate layer is very low compared with the core part, and structural foams can be treated as two-component beams that have skin layers of modulus equal to that of the matrix and a core with uniform modulus, the sandwich structure. Based on this assumption, the stiffness of structural foams is simply the sum of the stiffness of the core and skin parts. Hobbs[9] used several equivalent one-component beams to account for local variation in stiffness, and then obtained the deflection of structural foams under load. Tensile moduli of structural foams have not been as thoroughly studied, but are also related to morphology.[13–15] Throne gave an expression for the tensile modulus of a foamed sandwich structure in his book on thermoplastic foams.[13] Stokes and co-workers[14,15] gave the average elastic modulus for rectangular bars of structural foams. The average modulus of structural foams was related to the local elastic modulus.

Based on the amount of work available in the literature, several different approaches are used here to approximate the flexural and tensile moduli of polymer foams. In this study, HDPE foams were prepared using a compression molding technique to determine which models would best represent our measurements.

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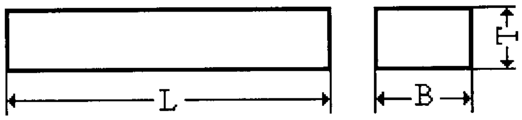

Figure 1 Sample dimensions for three-point bending tests.

EXPERIMENTAL

Polymer and sample preparation

Four HDPE foams with different molecular weights were used in this study. Foam plates with dimensions of 60 × 60 × 2.8–3.4 mm were obtained by compression molding. More details can be obtained in the first paper of this series.[1]

Skin thickness of the foams

Two specimens were used to measure the skin thickness for each condition. A Spot-Insight digital camera and software from Diagnostic Instruments were used to take pictures via an Olympus SZ-6 stereomicroscope. Quantitative measurements were performed using Image-Pro Plus image analysis software from Media Cybernetics (Silver Spring, MD). The procedure was as follows: (1) two photographs of the skin layers were taken for each specimen, (2) the distance between the bubbles closest to the skin and the surface was measured, (3) the distance measurements were averaged to obtain a value for the skin layer and the standard deviation was reported, and (4) the skin thickness of each specimen was divided by the thickness of the specimen to obtain the average ratio of skin to foam thickness.

Flexural measurements

Room temperature flexural properties were evaluated as a function of foam density using a Rheometrics Solids Analyzer RSA II with a transducer of 10 N. The samples were cut in rectangular shapes, as shown in Figure 1. The recommended dimensions for length (L), width (B), and thickness (T) are 52 mm, 8–10 mm, and 2.8–3.4 mm, respectively. A three-point bending fixture was used. The following conditions were also used: a temperature of 25°C, a strain rate of 0.004 $\sec^{-1}$, and a measuring time of 2 s. The stress (σ) and strain (ε) were determined by the following equations:[16,17]

$$\sigma = \frac{3L}{2BT^2} F \qquad (1)$$

$$\varepsilon = \frac{6T}{L^2} D \qquad (2)$$

where F is the measured force and D is the displacement of the transducer.

Tensile measurements

Room temperature uniaxial tension properties were evaluated as a function of foam density using an Instron 5565 tester with a 500 N load cell. The samples were cut in a type IV format according to ASTM D-638. Further details are given in the second paper of this series.[2]

Flexural modulus

Stiffness of materials

A beam of material in a three-point bending test is shown in Figure 2. The relation between curvatures at any point in the beam to the beading moment at that point is[5]

$$\frac{d^2y}{dx^2} = \frac{M}{(EI)} + \frac{d\beta}{dx} \qquad (3)$$

where y is the deflection from the neutral axis at any distance x, and M is the bending moment, usually expressed as a function of the distance x and the applied external loads. The product EI is the stiffness of the beam, and $d\beta/dx$ is the shear strain in the beam.

Stokes and coworkers[14,18] found that the effect of shear stress is proportional to the ratio of thickness over the square of the length of the specimen (T/L^2). This effect can become negligible by choosing specific dimensions. This is the case when the ratio L/T is higher than 18. Neglecting shear, eq. (3) simplifies to the well known expression[8,18–20]

$$\frac{d^2y}{dx^2} = \frac{M}{(EI)} \qquad (4)$$

If the beam is made of materials with different moduli, the stiffness of a symmetrical rectangular beam is[18]

$$(EI) = 2\int_0^{(1/2)T} E(y)y^2B(y)dy \qquad (5)$$

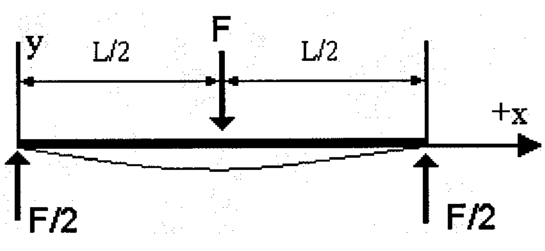

Figure 2 Force diagram of a three-point bending test.

TABLE I
Relationship Between Power Index of Eq. (12) and Poisson Ratio

ν_m	0.02	0.06	0.10	0.12	0.16	0.20	0.22	0.26
n	1.94	1.96	1.97	1.98	1.99	2.00	2.00	2.01
ν_m	0.30	0.32	0.36	0.40	0.42	0.46	0.49	0.495
n	2.01	2.00	2.00	1.98	1.97	1.95	1.93	1.93

where $E(y)$ and $B(y)$ are the elastic modulus and the width of the beam at a point y, respectively. The moment of inertia for a symmetrical beam made of the same material is defined as

$$I = 2\int_0^{(1/2)T} y^2 B(y)dy \tag{6}$$

If the beam is made of a material with a uniform modulus, the stiffness from eq. (5) becomes the product of the elastic modulus (E) and moment of inertia (I).

Integration of eq. (4) for a rectangular beam in three-point bending gives[9,19,20]

$$(EI)y_c = \frac{FL^3}{48} \tag{7}$$

where y_c is the absolute value of the central deflection. For a sample made of the same material, the moment of inertia for a rectangular cross-section (constant width and thickness) is obtained from eq. (5) as

$$I = 2\int_0^{(1/2)T} y^2 B(y)dy = BT^3/12 \tag{8}$$

Substituting eq. (8) into eq. (7) gives the modulus as

$$E = \frac{FL^3}{4y_c BT^3} \tag{9}$$

For three-point bending measurements, the flexural modulus obtained from the ratio of stress to strain [eqs. (1) and (2)] gives

$$E = \frac{\sigma}{\varepsilon} = \frac{3LF/2BT^2}{6TD/L^2} = \frac{FL^3}{4DBT^3} \tag{10}$$

Because the displacement of the transducer (D) measures directly y_c, eq. (10) is exactly the same as eq. (9). This means that for three-point bending experiments, the value of stiffness is the product of the average modulus and the moment of inertia. The actual stiffness from eq. (5) can be used to calculate the average modulus measured as

$$E_{ff} = \frac{2\int_0^{(1/2)t} E(y)By^2dy}{BT^3/12} = \frac{24\int_0^{(1/2)T} E(y)y^2dy}{T^3} \tag{11}$$

where E_{ff} is the modulus measured from a three-point bending experiment. This equation is similar to Khakhar and Joseph's approach.[21]

If the beam is made of a material with uniform density and modulus, the measured modulus is the elastic modulus. If the beam is made of materials with different moduli (sandwich-like structure), the modulus measured by the three-point bending experiment is not the elastic modulus, but an average flexural modulus that needs to be related to the structure of the beam. In the next section, some models are developed for foamed materials.

Models for uniform foams

For uniform foams, the modulus is constant at any point in the beam and is given by eq. (10). We have already discussed the models for the elastic moduli of two phase composites with spherical inclusions in the second part of this series.[2] The differential scheme and Moore's simple empirical equation (square power-law) best predicted the data in the range of void volume fractions (f) under study. The model is given by

$$\frac{E_f}{E_m} \approx (1-f)^n = \left(\frac{\rho_f}{\rho_m}\right)^n \quad (1.93 \le n \le 2.01) \tag{12}$$

where n is a power-law index, which is a function of the Poisson ratio of the matrix, as shown in Table I. In our study, the Poisson ratio for polyethylene is taken to be 0.34.[22] Eq. (12) is a general equation similar to the simple empirical equation of Moore,[4] which uses a value of 2 for n:

$$\frac{E_f}{E_m} \approx (1-f)^2 = \left(\frac{\rho_f}{\rho_m}\right)^2 \tag{13}$$

Flexural modulus models for structural foams

To determine the mechanical properties of a structural foam, it is necessary to take into account the effect produced by the non-uniform density across the foam section, the skin and core density. This is important

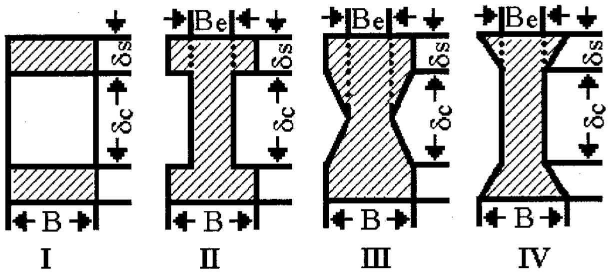

Figure 3 Equivalent beam cross sections for structural foams: (I) Cross-section of sandwich structure, (II–IV) Cross section of one-component beams; where δ_f is foam thickness, δ_c is thickness of core of foam, δ_s is thickness of skin layer, B is width of beam and B_e is equivalent width [$B_e = B(E_c/E_s)$].

because foams having the same overall density will have different mechanical properties based on the density difference between the skin and core, skin thickness, and other morphological parameters like cell size, density, and distribution.

For theoretical calculations, structural foams can be treated as two-component beams having skin layers of modulus equal to the matrix modulus, and a core section of uniform modulus, like the sandwich structure [Fig. 3(I)] of Gonzalez.[8] Hobbs[9] used equivalent one component beams to account for local variation in stiffness [Fig. 3(II–IV)]. Throne[6,7] assumed that for structural foams, the flexural modulus should be the average modulus through integration across the foam cross-section. We will now described each approach.

Gonzalez approach

Gonzalez[8] assumed that the intermediate zone is much smaller than the core zone. A structural foam structure is simplified as a three-layer panel like a sandwich structure [Fig. 3(I)]. The stiffness of such a beam $(EI)_f$ is the sum of the stiffness of the core and the skin:

$$(EI)_f = (EI)_c + (EI)_s \tag{14}$$

From eq. (11), the average flexural modulus is obtained:

$$E_{SF,f-1} = \frac{2\int_0^{(1/2)T} E(y)By^2dy}{B\delta_f^3/12}$$

$$= \frac{24\int_0^{(1/2)\delta_c} E_cy^2dy + 24\int_{(1/2)\delta_c}^{(1/2)\delta_f} E_my^2dy}{\delta_f^3}$$

$$= E_m - \left(\frac{\delta_c}{\delta_f}\right)^3(E_m - E_c) \tag{15}$$

where $E_{SF,f-1}$ is the average flexural modulus measured in three-point bending, E_c is modulus of core part, E_m is modulus of matrix, δ_c is thickness of core part, and δ_f is thickness of the structural foam.

According to this sandwich structure model, the density of the core is assumed to be uniform, and that of that of the skin layer is equal to the matrix. The density of the core (ρ_c) can be obtained via a mass balance:

$$\rho_f\delta_f = \rho_c\delta_c + 2\delta_s\rho_m = \rho_c\delta_c + (\delta_f - \delta_c)\rho_m \tag{16}$$

The normalized density of the core is expressed as:

$$\frac{\rho_c}{\rho_m} = 1 - \frac{\delta_f}{\delta_c}\left(1 - \frac{\rho_f}{\rho_m}\right) \tag{17}$$

$$= 1 - \frac{\delta_f}{\delta_c}[1 - (1 - f)] = 1 - \frac{\delta_f}{\delta_c}f$$

The normalized modulus of the core is obtained by substitution of eq. (17) into eq. (12).

$$\frac{E_c}{E_m} \approx \left(\frac{\rho_c}{\rho_m}\right)^n = \left(1 - \frac{\delta_f}{\delta_c}f\right)^n \quad (1.93 \le n \le 2.01) \tag{18}$$

The average normalized flexural modulus of beam is then obtained after substitution of eq. (18) into eq. (15) to give

$$\frac{E_{SF,f-1}}{E_m} = 1 - \left(\frac{\delta_c}{\delta_f}\right)^3 + \left(\frac{\delta_c}{\delta_f}\right)^3\left(1 - \frac{\delta_f}{\delta_c}f\right)^n$$

$$(1.93 \le n \le 2.01) \tag{19}$$

Hobbs approach

The stiffness of a structural foam beam $(EI)_f$ should be the same as that of a one-component equivalent beam $(EI)_{\text{model}}$, as shown in Figure 3:

$$(EI)_f = (EI)_{\text{model}} \tag{20}$$

Using eq. (12), several expressions of the average flexural modulus of a structural foam were obtained for the Hobbs models defining an equivalent width as $B_e = B(E_c/E_m)$ to give

$$E_{SF,f-II} = \frac{2\int_0^{(1/2)\delta_c} E_my^2B_edy + 2\int_{(1/2)\delta_c}^{(1/2)\delta_f} E_my^2Bdy}{B\delta_f^3/12} = E_m\left[(E_c/E_m)\frac{\delta_c^3}{\delta_f^3} + 1 - \frac{\delta_c^3}{\delta_f^3}\right] \tag{21}$$

$$E_{SF,f-III} = \frac{2\int_0^{(1/2)\delta_c} E_m y^2 \left(\frac{B - B_e}{\frac{1}{2}\delta_c} y + B_e\right) dy + 2\int_{(1/2)\delta_c}^{(1/2)\delta_f} E_m y^2 B dy}{B\delta_f^3/12} = \frac{E_c \delta_c^3}{4\delta_f^3} + \left(1 - \frac{\delta_c^3}{4\delta_f^3}\right) E_m \quad (22)$$

$$E_{SF,f-IV} = \frac{2\int_0^{(1/2)\delta_c} E_m B_e y^2 dy + 2\int_0^{(1/2)(\delta_f-\delta_c)} E_m \left(y + \frac{\delta_c}{2}\right)^2 \left(\frac{B - B_e\delta_s^3}{\delta_s} y + B_e\right) dy}{B\delta_f^3/12} = \frac{(E_c - E_m)}{4} \frac{(\delta_c + \delta_f)(\delta_c^2 + \delta_f^2)}{\delta_f^3} + E_m \quad (23)$$

According to the I-beam model [Fig. 3(II)], the density of the core of the structural foam is uniform and obtained from the following expression:

$$\rho_f\delta_f = 2\int_0^{(1/2)\delta_c} \rho_m \left[\frac{E_m(\rho_c/\rho_m)^n}{E_m}\right]^{1/n} dy + \rho_m(\delta_f - \delta_c)$$
$$= \rho_c\delta_c + \rho_m(\delta_f - \delta_c) \quad (24)$$

$$\frac{\rho_c}{\rho_m} = 1 - \frac{\delta_f}{\delta_c}\left(1 - \frac{\rho_f}{\rho_m}\right) \quad (25)$$

According to model III in Figure 3, the density of the core increases from the neutral axis to the intersection with the skin layer. The modulus increases linearly from the neutral axis to matrix modulus value at the intersection of core and skin sections:

$$\rho_f\delta_f = 2\int_0^{(1/2)\delta_c} \rho_m \left\{\frac{[E_m - E_m(\rho_c/\rho_m)^n]\frac{y}{\frac{1}{2}\delta_c} + E_m(\rho_c/\rho_m)^n}{E_m}\right\}^{1/n} dy + \rho_m(\delta_f - \delta_c) \quad (26)$$

To simplify and solve eq. (26), we set n equal to 2:

$$\rho_f\delta_f \approx 2\int_0^{(1/2)\delta_c} \rho_m \left\{[1 - (\rho_c/\rho_m)^2]\frac{y}{\frac{1}{2}\delta_c} + (\rho_c/\rho_m)^2\right\}^{1/2}$$
$$\times dy + \rho_m(\delta_f - \delta_c) \quad (27)$$

Performing integration gives

$$\rho_f\delta_f \approx \rho_m \frac{2}{3}\left[\frac{1 - (\rho_c/\rho_m)^3}{1 - (\rho_c/\rho_m)^2}\right]\delta_c + \rho_m(\delta_f - \delta_c) \quad (28)$$

$$\frac{\rho_c}{\rho_m} = \frac{1}{4}\frac{-3f + \delta_c/\delta_f + \sqrt{3(3f - \delta_c/\delta_f)(f - 3\delta_c/\delta_f)}}{\delta_c/\delta_f} \quad (29)$$

Because the core density is real and positive, the volume fraction of voids in structural foams (f) must satisfy this condition: $f \leq \frac{1}{3}\delta_c/\delta_f$. This indicates that this model cannot be used when the volume fraction of voids is higher than 0.32, even though the ratio of skin layers to foam thickness is very small (0.05).

For the modified I-beam model [Fig. 3(IV)], the core density is assumed to be uniform, but different from that of the skin layers. The modulus increases linearly, from the core modulus at the intersection of the core and layer parts, to the matrix modulus outside the skin:

$$\rho_f\delta_f = \rho_c\delta_c + 2\int_0^{\delta_s} \rho_m \left\{\frac{[E_m - E_m(\rho_c/\rho_m)^n]\frac{y}{\delta_s} + E_m(\rho_c/\rho_m)^n}{E_m}\right\}^{1/n} dy \quad (30)$$

Applying the same transformation as in eq. (26) gives

$$\rho_f\delta_f = \rho_c\delta_c + 2\int_0^{\delta_s} \times \rho_m\left\{[1 - (\rho_c/\rho_m)^2]\frac{y}{\delta_s} + (\rho_c/\rho_m)^2\right\}^{1/2} dy \quad (31)$$

and integration gives:

$$\rho_f\delta_f \approx \rho_c\delta_c + \frac{2}{3}\left[\frac{1 - (\rho_c/\rho_m)^3}{1 - (\rho_c/\rho_m)^2}\right]\rho_m(\delta_f - \delta_c) \quad (32)$$

$$\frac{\rho_c}{\rho_m} = \frac{1}{2}\frac{3\rho_f/\rho_m - 2 - \delta_c/\delta_f + \sqrt{\Delta}}{\delta_c/\delta_f + 2} \quad (33)$$

where

$$\Delta = 9(\delta_c/\delta_f)^2 + 6(\delta_c/\delta_f)(\rho_f/\rho_m) + 12(\delta_c/\delta_f) + 9(\rho_f/\rho_m)^2 + 12(\rho_f/\rho_m) - 12$$

Finally, the normalized moduli of the core were obtained for different models after substitution of the normalized density of eqs. (25), (29), and (33) into eq. (12). For the I-beam model, the normalized modulus is

$$\frac{E_c}{E_m} \approx \left(\frac{\rho_c}{\rho_m}\right)^n = \left(1 - \frac{\delta_f}{\delta_c} f\right)^n \quad (1.93 \le n \le 2.01) \quad (34)$$

For model III in Figure 3

$$\frac{E_c}{E_m} = \left(\frac{1}{4}\frac{-3f + \delta_c/\delta_f + \sqrt{3(3f - \delta_c/\delta_f)(f - 3\delta_c/\delta_f)}}{\delta_c/\delta_f}\right)^n \quad (1.93 \le n \le 2.01) \quad (35)$$

For the modified I-beam model

$$\frac{E_c}{E_m} = \left(\frac{1}{2}\frac{3\rho_f/\rho_m - 2 - \delta_c/\delta_f + \sqrt{\Delta}}{\delta_c/\delta_f + 2}\right)^n \quad (36)$$

Several expressions for the average normalized flexural modulus of different models were obtained through substitution of normalized core modulus of eqs. (34), (35), and (36) into eqs. (21), (22), and (23), respectively.

For the I-beam model

$$\frac{E_{SF,f-II}}{E_m} = \frac{\delta_c^3}{\delta_f^3}\left(1 - \frac{\delta_f}{\delta_c} f\right)^n + 1 - \frac{\delta_c^3}{\delta_f^3} \quad (37)$$

This equation is exactly the same as eq. (19) from Gonzalez's approach. This means that the I-beam model of Hobbs is equivalent to the sandwich structure model of Gonzalez.

For model III in Figure 3

$$\frac{E_{SF,f-III}}{E_m} = \left(\frac{1}{4}\frac{-3f + \delta_c/\delta_f + \sqrt{3(3f - \delta_c/\delta_f)(f - 3\delta_c/\delta_f)}}{\delta_c/\delta_f}\right)^n \frac{\delta_c^3}{4\delta_f^3} + 1 - \frac{\delta_c^3}{4\delta_f^3} \quad (38)$$

And for the modified I-beam model

$$\frac{E_{SF,f-IV}}{E_m} = \left[\frac{1}{4}\left(\frac{3\rho_f/\rho_m - 2 - \delta_c/\delta_f + \sqrt{\Delta}}{2(\delta_c/\delta_f + 2)}\right)^n - \frac{1}{4}\right] \times \frac{(\delta_c + \delta_f)(\delta_c^2 + \delta_f^2)}{\delta_f^3} + 1 \quad (39)$$

Throne's method

Throne[7] assumed that the modulus of structural foams in flexion is the average modulus of the beam obtained through integration across the foam cross-section:

$$E_{SF,f} = \int_0^{(1/2)\delta_f} E(y)dy \Big/ \int_0^{(1/2)\delta_f} dy \quad (40)$$

Throne assumed that the density of structural foams increased continuously from the neutral axis to the surface of the sample even if it is very difficult to measure the local density of structural foams. Here we use Throne's basic method to calculate the modulus with several simple models.

The sandwich structure model [Fig. 3(I)] assumes that the structural foam consists of a uniform core and uniform skin layers. The average modulus is calculated using eq. (40):

$$E_{SF,f-I} = \left(\int_0^{\delta_c} E_c dy + 2\int_0^{\delta_s} E_m dy\right) \Big/ \int_0^{\delta_f} dy = [E_c\delta_c + E_m(\delta_f - \delta_c)]/\delta_f \quad (41)$$

Substitution of eq. (34) into eq. (41) gives

$$\frac{E_{SF,f-I}}{E_m} = \left(1 - \frac{\delta_c}{\delta_f}\right) + \frac{\delta_c}{\delta_f}\left(1 - \frac{\delta_f}{\delta_c} f\right)^n \quad (1.93 \le n \le 2.01) \quad (42)$$

The I-beam model also assumes that structural foams consist of a uniform core and uniform skin layers. The average modulus is thus the same as that of the sandwich structure model.

On the other hand, model III assumes that structural foams consist of uniform skin layers with a core in which the modulus increases linearly from a neutral axis to matrix the intersection between the core and skin layers. The average modulus using eq. (40) is

$$E_{SF,f-III} = \left[\int_0^{(1/2)\delta_c}\left(\frac{E_m - E_c}{\frac{1}{2}\delta_c} y + E_c\right)dy + \int_{(1/2)\delta_c}^{(1/2)\delta_f} E_m dy\right] \div \int_0^{(1/2)\delta_f} dy = E_m\left(1 - \frac{\delta_c}{2\delta_f}\right) + E_c\frac{\delta_c}{2\delta_f} \quad (43)$$

The normalized average modulus was obtained after placing into eq. (43) the core modulus from eq. (35):

$$\frac{E_{SF,f-III}}{E_m} = \left[\frac{1}{4}\,\frac{-3f + \delta_c/\delta_f + \sqrt{3(3f - \delta_c/\delta_f)(f - 3\delta_c/\delta_f)}}{\delta_c/\delta_f}\right]^n \times \frac{\delta_c}{2\delta_f} + \left(1 - \frac{\delta_c}{2\delta_f}\right) \quad (44)$$

The modified I-beam model assumes that structural foams consist of a uniform core and skins, and that the modulus increases linearly from a core modulus at the intersection between core and skin to the matrix modulus at the surface:

$$E_{SF,f-IV} = \left[\int_0^{\delta_s} E_c dy + 2\int_0^{\delta_s}\left(\frac{E_m - E_c}{\delta_s}\,y + E_c\right)dy + \right] \div \int_0^{\delta_f} dy = E_c\left(\frac{1}{2} + \frac{\delta_c}{2\delta_f}\right) + E_m\left(\frac{1}{2} - \frac{\delta_c}{2\delta_f}\right) \quad (45)$$

The normalized average modulus is obtained after placing into eq. (45) the core modulus from eq. (36):

$$\frac{E_{SF,f-IV}}{E_m} = \left(\frac{1}{2} + \frac{\delta_c}{2\delta_f}\right)\left(\frac{1}{2}\,\frac{3\rho_f/\rho_m - 2 - \delta_c/\delta_f + \sqrt{\Delta}}{\delta_c/\delta_f + 2}\right)^n + \left(\frac{1}{2} - \frac{\delta_c}{2\delta_f}\right) \quad (46)$$

Tensile modulus models of structural foams

For structural foams with rectangular cross-sections, the tensile modulus can be obtained by assuming a constant strain in the skin and core sections via this model:[13]

$$E_{f,T}\delta_f = \int_0^{\delta_f} E(y)dy \quad (47)$$

where $E_{f,T}$ is tensile modulus of the structural foam, δ_f is thickness of beam, and $E(y)$ is the local modulus at y. When the beam is symmetrical with respect to the neutral axis, the average tensile modulus is obtained by

$$E_{f,T} = \frac{2\int_0^{\delta_f/2} E(y)dy}{\delta_f} \quad (48)$$

This equation is similar to eq. (40) for the flexural modulus. The normalized modulus becomes

$$\frac{E_{f,T}}{E_m} = \frac{2\int_0^{\delta_t/2} \frac{E(y)}{E_m}\,dy}{\delta_f} \quad (49)$$

Sandwich structure model

Using eq. (49), the normalized tensile modulus for the sandwich structure foam model is

$$\frac{E_{f,T}}{E_m} = \frac{2\int_0^{(\delta_c/2)} \frac{E_c}{E_m}\,dy + 2\int_{(\delta_c/2)}^{(\delta_f/2)} \frac{E_m}{E_m}\,dy}{\delta_f} = \frac{E_c}{E_m}\,\frac{\delta_c}{\delta_f} + \left(1 - \frac{\delta_c}{\delta_f}\right) \quad (50)$$

The average normalized tensile modulus of the structural foams as a function of the normalized density is obtained after the substitution of eq. (18) into eq. (50):

$$\frac{E_{SF,T-I}}{E_m} = \left(1 - \frac{\delta_f}{\delta_c}f\right)^n \frac{\delta_c}{\delta_f} + \left(1 - \frac{\delta_c}{\delta_f}\right) \quad 1.93 \le n \le 2.01 \quad (51)$$

which is similar to eq. (42).

Other models

Because the I-beam model is actually the same as the sandwich structure model, and model III is only valid for very low volume fractions, these models will not be discussed further. On the other hand, the modified I-beam model of eq. (49) can be used to give

$$\frac{E_{SF,T-IV}}{E_m} = \left[\int_0^{\delta_s} \frac{E_c}{E_m}\,dy + 2\int_0^{\delta_s} \frac{(E_m - E_c)y/\delta_s + E_c}{E_m}\,dy + \right] \div \delta_f = \frac{E_c}{E_m}\left(\frac{1}{2} + \frac{\delta_c}{2\delta_f}\right) + \left(\frac{1}{2} - \frac{\delta_c}{2\delta_f}\right) \quad (52)$$

The normalized average modulus was obtained after placing into eq. (52) the core modulus from eq. (36):

$$\frac{E_{SF,T-IV}}{E_m} = \left(\frac{1}{2} + \frac{\delta_c}{2\delta_f}\right)\left(\frac{1}{2}\,\frac{3\rho_f/\rho_m - 2 - \delta_c/\delta_f + \sqrt{\Delta}}{\delta_c/\delta_f + 2}\right)^n + \left(\frac{1}{2} - \frac{\delta_c}{2\delta_f}\right) \quad (53)$$

RESULTS AND DISCUSSION

Thickness of HDPE foam skins

With the compression molding process, the foams usually have a very thin skin layer on each side. The

TABLE II
Ratio of Skin Thickness to Foam Thickness (%)

ACA (%)	Skin	J60-1700-173	A60-70-162	G60-110	HBW555Ac
1.0	Upper	—	2.18 ± 0.67	2.31 ± 0.25	1.96 ± 0.32
	Lower	—	2.31 ± 0.31	2.45 ± 0.34	1.95 ± 0.32
1.5	Upper	2.59 ± 0.35	1.50 ± 0.13	2.10 ± 0.14	1.98 ± 0.44
	Lower	2.51 ± 0.34	2.11 ± 0.29	2.06 ± 0.39	1.87 ± 0.23
2.0	Upper	3.02 ± 0.50	1.80 ± 0.24	2.52 ± 0.24	2.41 ± 0.43
	Lower	2.62 ± 0.23	1.79 ± 0.22	1.40 ± 0.19	2.00 ± 0.26
2.5	Upper	2.48 ± 0.25	1.66 ± 0.11	1.59 ± 0.15	1.71 ± 0.07
	Lower	2.22 ± 0.19	1.50 ± 0.13	1.67 ± 0.29	1.84 ± 0.20
3.0	Upper	2.17 ± 0.42	1.55 ± 0.16	1.83 ± 0.32	1.54 ± 0.11
	Lower	2.27 ± 0.22	1.53 ± 0.13	1.82 ± 0.31	1.61 ± 0.14

ratio of the skin thickness to the foam thickness is presented in Table II for each condition tested. From Table II, it can be seen that the average skin thickness is small, symmetrical, and almost constant for each polyethylene. The average values of both skins for J60-1700-173, A60-70-162, G60-110, and HBW555Ac at different blowing agent concentrations are: 5.0 ± 0.6%, 3.5 ± 0.4%, 4.0 ± 0.5% and 3.8 ± 0.5%, respectively. This gives a total average thickness of 4.0 ± 0.5% for all foams, and an average relative core thickness (δ_c/δ_f) of 96.0 ± 0.5%.

Flexural properties

The flexural modulus of HDPE foams as a function of normalized density is shown in Figure 4. Because the foam modulus is related to the matrix modulus, the normalized modulus (ratio of foam modulus to unfoamed polymer matrix modulus) and normalized density (ratio of foam density to unfoamed polymer matrix density) are used to analyze the relationship between modulus and density in order to eliminate the relative effect of the unfoamed polymer matrix on the foam. The normalized modulus as function of normalized density is shown in Figure 5. This transformation produces a single curve.

Comparison of flexural models

All of the models described contain simplifying assumptions on the structure and the distribution of voids that are likely unrealistic. Their validity must be judged, at least partially, in terms of how closely they predict the experimental data between the normalized modulus and normalized density.

Structural foam models were first used to evaluate the effect of skin thickness on flexural modulus. In each case, the calculations were made using a ν_m value of 0.34, as described earlier. The average deviations of each model at different skin thickness ratios are presented in Table III. Values for δ_c/δ_f were changed between 95% and 97% to determine the effect of this parameter. The results show that the minimum deviation is around 96%, which confirms the optical microscopy measurements. For this reason, a value for δ_c/δ_f of 0.96 will be used in the following discussion.

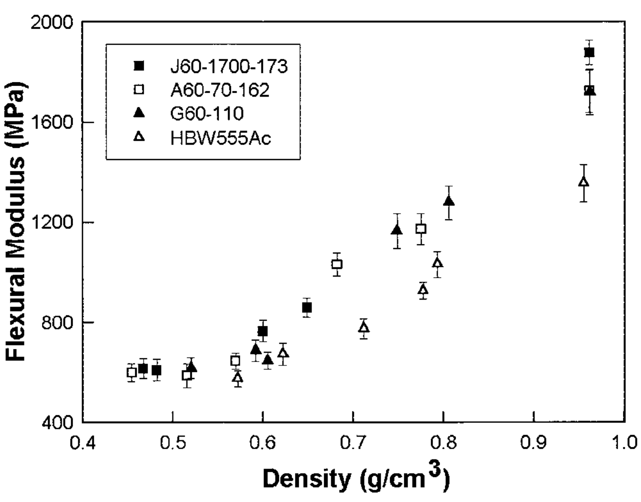

Figure 4 Flexural modulus as a function of density.

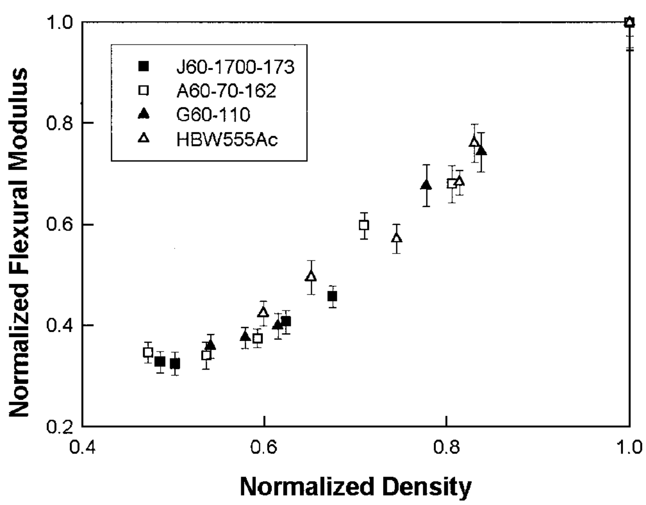

Figure 5 Normalized flexural modulus as a function of normalized density.

TABLE III
Average Deviation of Different Models for Normalized Flexural Modulus

Equation	δ_c/δ_f 97.0%	96.5%	96.0%	95.5%	95.0%
19	6.5	6.0	5.8	5.8	6.1
37	6.5	6.0	5.8	5.8	6.1
39	8.9	8.5	8.0	7.6	7.3
42	11.2	11.0	10.9	10.8	10.6
46	11.7	11.6	11.6	11.5	11.5

Gonzalez and hobbs approaches

Figure 6 shows a comparison of the normalized modulus as a function of the normalized density for both the Gonzalez and the Hobbs approach. It can be seen that model III does not fit the data, and when the volume fraction of the structural foam reaches $\frac{1}{3}\delta_c/\delta_f$, the core modulus at the neutral axis equals zero. The other models seem to fit the data reasonably well. The average deviations were found to be 5.8%, 5.8%, and 8.0% for the model of Gonzalez, the I-beam model of Hobbs, and the modified I-beam model of Hobbs respectively.

Throne's method

Figure 7 shows a comparison of the normalized modulus as function of the normalized density using Throne's method. From Figure 7, it can be seen that these predictions seem to underestimate the experimental data. Even though model III seems to fit the experimental data better, this model has the same limitation described in the previous section. The average deviations were found to be 10.9%, 10.9%, and 11.6% for models I, II, and IV of Figure 3. These higher deviations are probably related to the fact that Throne's method only considers the effect of density on modulus, but does not consider the effect of density on stiffness as a whole. For thick skin layers and high volume fraction of voids, the difference is more obvious.

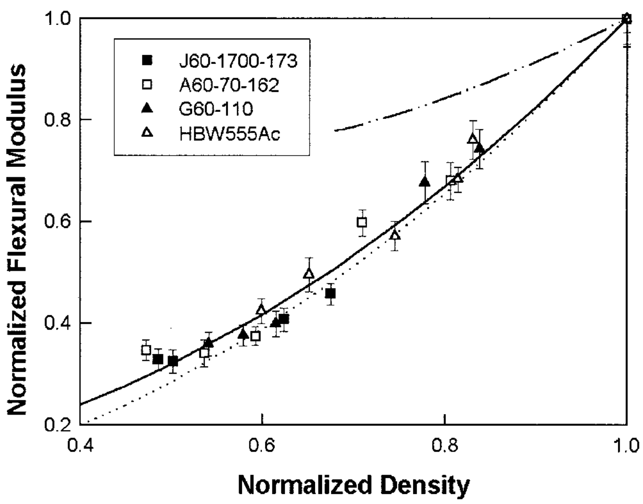

Figure 6 Comparison of the normalized flexural modulus as function of normalized density for Gonzalez and Hobbs approaches: (–··–··–) model III of Hobbs, (—) model of Gonzalez and I-beam model of Hobbs, (– – –) modified I-beam model of Hobbs.

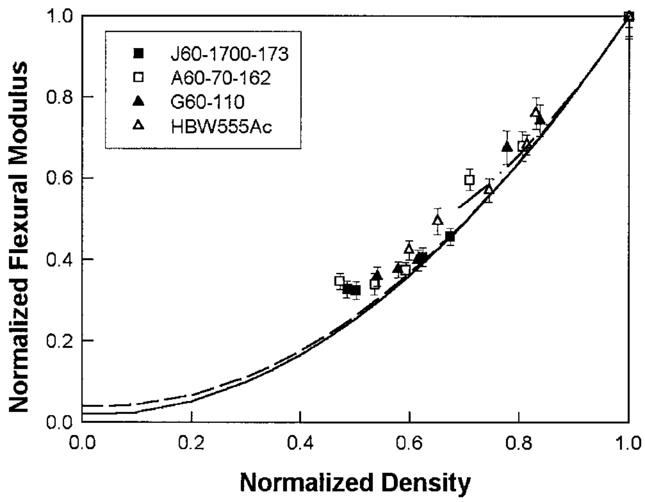

Figure 7 Comparison of the normalized flexural modulus as a function of normalized density for Throne method: (–··–··–) model III, (– – –) model I and II, (—) model IV of Figure 3.

Comparison between uniform and structural foam models

Figure 8 shows the normalized modulus as function of normalized density using the differential scheme for uniform foam, Moore's empirical square law relation,

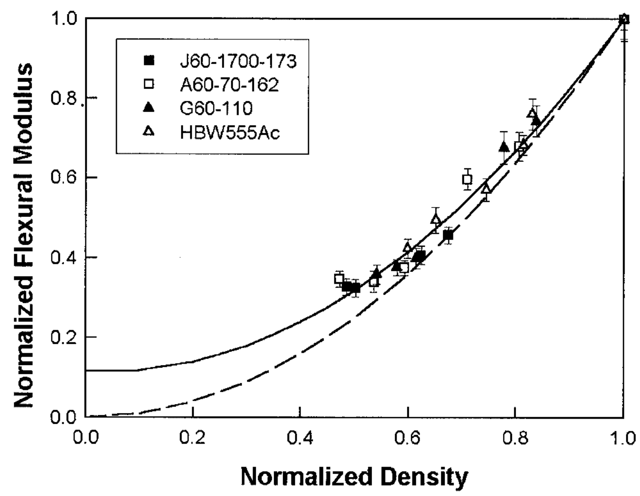

Figure 8 Comparison of the normalized flexural modulus as a function of the normalized density for uniform and structural foams: (– – –) Differential scheme, (—) Gonzalez approach and I-beam model of Hobbs.

the Gonzalez approach, and the I-beam model of Hobbs. The average deviations are 12%, 12%, 5.8%, and 5.8% respectively. These results indicate that skin layers, even very small ones, have an important effect on the flexural modulus of polymer foams.

Tensile modulus

We have also compared different approaches for the tensile modulus for uniform foams.[2] The differential scheme and Moore's empirical square power-law models were found the best to represent the tensile moduli of our closed-cell HDPE foams with an average deviation of 7.9% for both models. We will now evaluate whether thin skins have an effect on tensile modulus.

Table IV shows the average deviations of the different tensile modulus models with different thicknesses of skins ($\nu_m = 0.34$). It can be seen that the differences for values of δ_c/δ_f between 95 and 97% is almost insignificant. Once again, the minimum deviation is around $\delta_c/\delta_f = 96\%$.

Figure 9 shows excellent agreement between the models and the experimental data. The average deviations are 7.3%, 7.7%, 7.9%, and 7.9% for the differential scheme of a sandwich, the differential scheme of the modified I-beam structure, the differential scheme for uniform foam, and Moore's empirical model for uniform foam respectively. For structural foam models, the elastic modulus does not reach zero when the void volume fraction reaches one because these structural foam models consist of skin layers and a foamed core. The void volume fraction cannot reach unity even though the core foam's void volume fraction reaches 1 (two skin layers will remain to support the load). The difference in the average deviation for these three models is less than 0.7%. This means that very thin skin layers for HDPE foams have a very small effect on the tensile modulus.

CONCLUSIONS

Flexural and tensile moduli of uniform and structural closed-cell HDPE foams were measured and compared to different models from the literature.

For the flexural modulus, the differential scheme and Moore's empirical square power-law models for uniform foams were found to deviate around 12% from the measurements. On the other hand, Gonzalez's sandwich structure model and Hobbs I-beam models for structural foams were found to better fit the data with an average deviation of 5.8%. This reduced deviation indicates that, for the flexural properties of polymer foams, even very thin skins have a definite effect on the modulus. We also found that Throne's method underpredicted our data.

For the tensile modulus, models for uniform and structural foams were also investigated. For structural foam models, it was found that a combination of the differential scheme with the sandwich structure and a combination of the square power-law model with the sandwich structure gave the smallest average deviation at 7.3%. For uniform foam models, the differential scheme and Moore's empirical square power-law models also gave a good prediction, with an average deviation of 7.9%. The difference between structural and uniform foams is very small (0.7%). This seems to indicate that very thin skins have little effect on tensile modulus.

TABLE IV
Average Deviation of Different Models for Normalized Tensile Modulus

	δ_c/δ_f				
Equation	97.0%	96.5%	96.0%	95.5%	95.0%
51	7.4	7.4	7.3	7.4	7.4
53	7.7	7.7	7.7	7.6	7.6

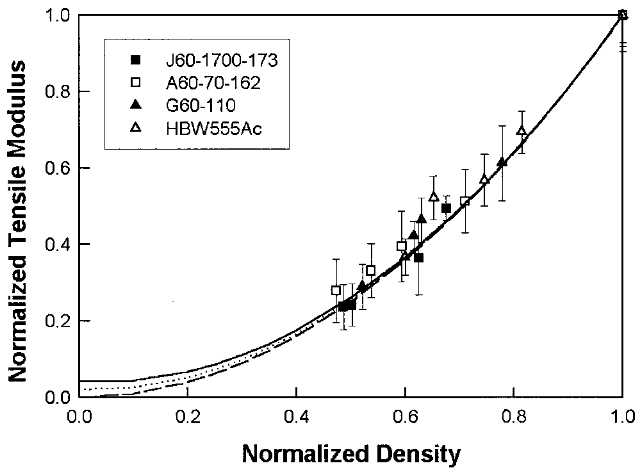

Figure 9 Comparison of the normalized tensile modulus as a function of the normalized density for tensile models: (– – – –) differential scheme and Moore's empirical equation, (- - - - - - - - -) eq. (53) for structural foams, (—) eq. (51) for structural foams.

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